MOUNTAIN MEASUREMENTS.

III.—TRIGONOMETRIC METHODS.

By J. C. Barnett.

In former papers the measurement of heights by means of barometers, hypsometers, and thermometers was described, and now, in conclusion, it is intended to sketch briefly the methods used by the officers of the Ordnance Survey, first for fixing the exact position of the mountains, and second for determining their elevation above sea level. The principle is simple, but the working, so as to obtain perfectly reliable results, is difficult in the extreme, and requires great care, patience, manipulative skill, and the power to solve some rather abstruse problems in mathematics.

If we were walking on the sea shore, and wished to determine the distance of a ship riding at anchor in the bay, it would not be necessary to pay a visit to the vessel, as its distance could be calculated from certain data easily obtainable on the shore. We would measure carefully a line on the beach, say 200 feet long, and place a staff at each end of it. Then we would, by means of a theodolite or compass, determine the angles which lines drawn from each end of our base to the ship would make with the measured base itself. This drawn to scale would show the relative positions of the base line and ship, and with care the ship's distance might be determined *geometrically*. But the problem may be solved also by trigonometry. The sides and angles of any triangle are directly proportional to each other, and so the dimensions of any triangle can be calculated whenever one side and two angles, or two sides and one angle, are known. It is plain, therefore, that if an accurate base line has been measured, and the angular distance of a point from each end of it has been determined, then the distance of that point from each end of the base line can be calculated by the use of trigonometrical formulæ. These calculated distances may be used as base lines for the determination of the exact positions and distances of other points, and so in this way the whole country may be covered with a vast network of triangles, and the relative positions of each of the stations ascertained with exceeding accuracy.

The accuracy of these results depends on the perfect measurement of the base line and the exact determination of the various angles by the theodolite. Both of these prove to be much simpler in theory than they turn out to be in practice. The first requisite, then, is a base line which must be absolutely straight, perfectly level, and of sufficient length—the longer the better. As the earth is spherical, "perfectly level" means that every part of the line must be at right angles to the direction of a line to the centre of the earth, or, in other words, to the plumb line. When the base has been aligned, it is measured by rods of metal, wood, or glass, which are not permitted to touch each other, lest by the placing of them the perfect alignment of the rods might be disturbed. The spaces between each pair are measured by microscopes and added to obtain the true length. As, however, these rods expand by heat and contract by cold, a certain temperature has to be fixed upon, and all measurements by rods reduced to what they would be at that temperature. The temperature chosen is 62° Fah.

General Roy measured the first base line in England on Hounslow Heath in 1783, and used wooden rods, each 20 feet long, cut out of an old Riga mast, but he found that these rods changed so much in length through the effect of moisture that he abandoned them for glass rods. In proof of the wonderful accuracy of General Roy's party, it may be stated that this base line was measured three times in 1783 with wooden rods, steel chain, and glass tubes, and again in 1791 with a steel chain, and though the length of the base was five miles the greatest difference between the measurements was under six inches.

In 1827 Major-General Colby invented his compensating bars, consisting of two bars, one of brass and one of iron, so combined as to eliminate changes of length due to variations of temperature. The two bars are fixed together in the middle by a steel cylinder, but free to expand at the ends.

Joining the free ends of the two bars are two steel tongues, so fixed by pivots rivetted into the bars that they are capable of free movement. These tongues extend beyond the bars, and the distance between their two points is 10 feet. If the upper rod of brass alone expanded, then the points would be brought nearer by the tongues revolving on their pivots, while, if the iron rod alone expanded, then the points would be pushed farther apart. As, however, the two metals expand different amounts for the same degree of heat, the rods have been so arranged that the unequal expansion acts in such a way as to keep the points at an unvarying distance, whatever be the variations of temperature to which the bars are subjected. A difficulty arises here. As brass and iron have different capacities for heat, and their surfaces present different powers of radiation and absorption, it follows that the same amount of heat will produce different degrees of temperature in each of the metals. Now, as it is essential that Colby's bars should be at exactly the same temperature, the surfaces of these bars have been coated with varnish until, by experiment, it has been proved that the bars increased and diminished by the same amount when similarly exposed.

Drummond's base line on the level shore of Loch Foyle was measured in 1827 by Colby's bars. This base line is 7$\frac{4}{7}$ miles, and the probable error of measurement is not more than 2 inches. When the Salisbury base line of 1794 was measured from triangles, originating from the Loch Foyle base line at a distance of 350 miles, the calculated length differed from the measured length by only 5 inches.

From what has been said, it must be granted that those engaged in our Ordnance Survey could measure a base line with marvellous accuracy, and this having been obtained the work of triangulation commences. Here, again, not a few difficulties meet us at the outset. First, as the earth is globular, lines drawn upon its surface are arcs of circles, and therefore differ essentially from those drawn upon a plane surface. This would matter little were the triangles small, but in triangulation, in order to avoid the errors that would arise from many angular measurements, the triangles are made as large as possible. In a plane triangle the value of the three angles amounts to 180°, but in a spherical triangle the amount is more. Thus, with a triangle having sides 13 miles long, the excess would amount to one second of arc, and this spherical excess has to be allowed for in calculating the lengths of the sides. There are several ways of solving these triangles, but of late the favourite method is that introduced by Legendre, which is applicable to all triangles drawn on spherical surfaces, whose sides are small compared with the radius of the sphere upon which they are drawn. It may be expressed thus: Diminish each of the angles by one-third of the spherical excess, thus reducing the sum to 180°, and then treat as plane triangles.

If the earth were a perfect sphere, the measurement then of these spherical triangles would present no great difficulty after the observations had been made and checked, but as the earth is not a perfect sphere, but an oblate spheroid flattened at the poles and slightly compressed at the equator, the problem becomes one of some difficulty, and entails the calculation of the radius of curvature at various parts of the earth's surface, by no means a simple task. The difference between the lengths of the polar and equatorial diameters is 26 miles, and between the two equatorial diameters 1524 feet. Again, as the three stations of a great triangle will likely be at different heights above the earth's surface, it becomes necessary to reduce all to sea level before measuring the length of the sides.

In measuring the heights of distant stations for the purpose of reducing all to sea level, not only must we allow for the curvature of the earth, which makes the height appear less than it really is, but also for the refraction of the light rays in their passage from the distant object to the eye, which tends to make the height appear greater. As the amount of refraction depends on the state of the atmosphere, and is constantly varying, sometimes in a most extraordinary way within a few hours, it becomes a matter of great difficulty to apply corrections for refraction. Again, the plumb line which theoretically always points to the centre of the earth, is often deflected by the presence of mountains, or rocks of high specific gravity, or even by underground caverns, and errors of some magnitude, that are highly difficult to discover and eliminate, creep thus into calculations. Besides all these sources of error, there are those arising from defects in the instruments used, and those due to the temperament of the observers, both of which it is impossible to eliminate, but must be allowed for. It will thus be seen that though in theory the determination of the true position of places on the earth's surface is simple enough, yet to obtain accurate results time, care, judgment, patience, and high mathematical skill are required.

To determine the heights of mountains by trigonometry involves nothing more than is necessary to determine the true position of distant points, viz., a base line and two angles. Measure off a base of some length between two points in the same straight line as the mountain top, then with the theodolite measure the angular elevation of the mountain top from each point. Lines drawn from each end of the base at the inclinations observed would meet at the summit, and a perpendicular dropped from this point to the base produced would represent the height required. This height may be calculated by the application of trigonometrical formulæ. Heights are frequently measured by the theodolite and staff, and though the method involves much labour and care, excellent results have been obtained by it. The distant mountains of moon-land have had their elevations determined from observations of the lengths of their shadows.

The Survey of Great Britain has just been completed, but already, owing to the length of time since it was begun, calls loudly for revision. This Survey gave employment to a staff of 3200 men, of whom 380 were engineers. Ten thousand eight hundred sheets have been produced in scales varying from 10·56 feet to $\frac{1}{10}$ inch to the mile, the former being a representation of the surface as it would appear from a height of 750 feet, and the latter from a height of 1670 miles. In the Survey 29 triangles had sides of 90 miles length, and 11 had sides measuring 100 miles. We may well be proud of our Ordnance Survey and the men who have so successfully carried it out, for the scope of the undertaking is greater than that of any other Government, and the mode and style of the work, whether viewed from a scientific, artistic, or utilitarian stand-point, will hold a first place among the great Geodetic Surveys of the world.

THE TEACHING OF THE HILLS.

THE everlasting hills, their country's pride,
  In silence dominate the crowded plains,
  Where busy mortals toil for trivial gains,
Remembering not the lessons hills provide,
Mute witnesses of Time's remorseless tide,
  Sweeping away frail man as he attains
  Some object cherished long, for which he strains
With utmost power; the hills such folly chide.
  Though fiercely rent by many a stormy blast,
Majestic their gigantic forms appear,
  The links sublime with ages of the past;
These heights, with caution due, the mountaineer
  Ascends, that he afar his gaze may cast
And rise in thought beyond earth's narrow sphere.

ADAM SMAIL.

13 CORNWALL STREET,
Edinburgh.