CURVATURE AND REFRACTION:

Notes as to Observations from The Blue Hill, Aberdeen.

By G. Gordon Jenkins, C.E.

In the *Cairngorm Club Journal* for July, 1895 (Vol. I., pp. 265-7), there is an excellent article by Rev. Robert Semple dealing with the "Distance of the Visible Horizon," which included a table as the basis for calculations. In this table, however, no allowance was made for refraction. Professor Rankine, who ought to be an authority, states that the effect of refraction varies from $\frac{1}{10}$th to $\frac{1}{2}$ of the allowance for curvature of the earth, and that $\frac{1}{8}$th would be a general average. Scientific authorities generally agree that this is the nearest approach to an average allowance for atmospheric refraction, and this is confirmed in the "Account of the Principal Triangulation of the Ordnance Trigonometrical Survey of Great Britain and Ireland" by Lt.-Colonel H. James, R.E., 1858. The effect of the earth's curvature is to make a distant object appear *lower* than it would be seen to be were the earth flat. On the other hand the line of sight—that is, the line along which the light proceeds from the object looked at to the eye of the observer—is not perfectly straight, being rendered slightly concave downward by the refracting action of the air, thus making the object looked at appear *higher* than it really is, and consequently visible at a greater distance than if the sight line were a straight one. To this extent, varying with the density and temperature of the air, the effect of the earth's curvature is partly neutralised.

Refraction is greatest early in the morning, diminishes till 10 a.m., is nearly constant till 4 p.m., and then begins to increase. Great variation sometimes occurs over ground which is passing from light to shade, or *vice*

versa. It will thus be noted that with such a fluctuating quantity, as to the effects of which there is a variety of opinion, it is impossible to tabulate what allowance has to be made except as an approximation. While this is so, refraction cannot be ignored altogether; and this article is based on the assumption that, as a fair average, in estimating the distance of the visible sea horizon one-sixth is to be deducted from the allowance for the earth's curvature.

With this allowance for refraction, the heights corresponding to various distances of the visible sea horizon may be calculated from the formula :—

$$\text{Height in feet} = 0.556 \times (\text{distance in statute miles})^2$$

which gives the results tabulated at the end of this article.

In order to work out the combined effect of both curvature and refraction on a different allowance from that just assumed, one has only to square the distance in miles and multiply by 0.611 in the case of $\frac{1}{12}$th, by 0.600 for $\frac{1}{10}$th, by 0.572 for $\frac{1}{7}$th, by 0.530 for $\frac{1}{5}$th, and by 0.500 for $\frac{1}{4}$th, instead of by 0.556 as in the above formula.

To demonstrate in a practical way the effect of curvature and refraction, it is proposed to apply the figures in the table to some of the hills noted in the article on the Blue Hill by the late Dr. Alexander Cruickshank and the late Mr. Alexander Copland which appeared in the first volume of the *C.C.J.* (pp. 29-45).

Morrone may be selected to illustrate how the table is applied. To begin with, in order to ascertain whether a certain hill is visible or not, one has to draw a straight line on a map from the point of vision to the selected hill, and note the prominent heights lying between. Between the Blue Hill and Morrone there are two such intervening heights, viz. — Baudy Meg and Knockie Branar; and it will be found that the straight line from the Blue Hill to Morrone passes over the first prominent height—viz., Baudy Meg—a little to the north of its summit at a point about 1510 feet above sea level, and over the second height—viz., Knockie Branar—a little

to the south of its summit at a point about 1900 feet above sea level. All the data can now be stated :—

1. The Blue Hill, the point of observation, 480 feet high, to top of Cairn.
2. Baudy Meg, at a height of 1510 feet and 27 miles distant.
3. Knockie Branar, at a height of 1900 feet and 33¼ miles distant.
4. Morrone, 2819 feet high and 50 miles distant.

The next step is to reduce the levels of Baudy Meg, Knockie Branar, and Morrone relatively to a line drawn through the top of the Blue Hill tangential to the surface of the earth. This is done by deducting from the height of each hill the height given in the table corresponding to its distance from the Blue Hill, and then deducting the height of the Blue Hill. Thus :—

	Baudy Meg. Feet.	Knockie Branar. Feet.	Morrone. Feet.
Height - - - -	1510	1900	2819
Less height in table - -	405	615	1390
	1105	1285	1429
Less height of Blue Hill -	480	480	480
Reduced level - - -	625	805	949

Then, by proportion, we ascertain that the reduced level of the line of vision from the Blue Hill over Baudy Meg will, at Knockie Branar, be :—

$$\frac{33\frac{1}{4} \text{ miles}}{27 \text{ ,,}} \times 625 = 769 \text{ feet.}$$

This is 36 feet lower than the reduced level of Knockie Branar, and therefore the observer on the Blue Hill will see the upper 36 feet of Knockie Branar over the top of Baudy Meg, and Knockie Branar is therefore the dominant height.

In a similar manner we find that the reduced level of the line of vision from the Blue Hill over Knockie Branar will, at Morrone, be :—

$$\frac{50 \text{ miles}}{33\frac{1}{4} \text{ ,,}} \times 805 = 1211 \text{ feet.}$$

But the reduced level of Morrone, as above noted, being 949 feet, Morrone is not visible by 262 feet.

An examination of the diagram which accompanies this paper should make these calculations quite clear.

In the Cruickshank-Copland panorama and relative list, not only is Morrone said to be visible from the Blue Hill, but there are four other hills in the gap between Lochnagar and Beinn Bhrotain, viz :—Creag Liath, Carn Fiaclan, Carn Cloich Mhuilinn, and Carn na Drochaide—which are incorrectly noted as visible. The last is evidently drawn out of its true position, which is almost exactly in a line to the summit of Beinn Bhrotain, instead of (as shown) considerably to the south of that line, and appearing as a sky-line hill, which it is not.

Attention may here be incidentally directed to several instances of what appear to be inexactitudes in the drawing of the panorama referred to. Cock Cairn is shown as lying to the left, or south, of a line to the summit of Mount Keen, whereas it lies to the right, or north, of that line. Besides, Cock Cairn does not appear as a sky-line hill from the Blue Hill (as mistakenly shown on the panorama), wanting about 60 feet in height to be seen over the north shoulder of Mount Keen. But the Gathering Cairn, a prominent spur stretching out northward from Braid Cairn (shcwn on the panorama, but not named), is a sky-line hill.

According to calculations deduced from the table subjoined, Creag Liath (over the north shoulder of Cairn Leughan) is invisible by 76 feet; Carn Fiaclan (over the south shoulder of Cairn Leughan) by 107 feet, and Carn Cloich Mhuilinn (exactly over the summit of Pananich Hill) is not seen by as much as 813 feet, and even if the earth were flat, only 71 feet of it would be visible.

If these calculations are correct as an average of the allowance to be made for refraction, one is driven to the conclusion that there is only one hill-top visible beyond the Pananich-Cairn Leughan ridge (and one not noted in the Cruickshank-Copland list), viz., Meall Coire na Saobhaidhe, quite a distinct peak lying to the north of Lochnagar, 168 feet of which (over the summit of Clochan Yell) is visible from the Blue Hill. At any rate, if there be any other distant hill visible beyond the

Pananich ridge in the gap between Lochnagar and Beinn Bhrotain, it is up to members of the Cairngorm Club to rope in the lost mountain and demonstrate its visibility. It occurred to me that some mountain near the head of the Geldie might possibly come into view, but that does not seem at all likely, because I find that the two most prominent mountains in that locality—Carn Ealar and Carn an Fhidheleir Lorgaidh—would require to be at least 1122 and 1682 feet higher respectively to come into view.

The following is a note of the extent visible from the Blue Hill of a few of the sky-line hills, of which little more than the summits are seen :—

1. Mount Keen (over north shoulder of Braid Cairn ridge) . . 106 ft.
2. ,, north shoulder (over Cock Cairn and Gathering Cairn ridge) . . . . . 65 ,,
3. Meall Coire na Saobaidhe (over Clochan Yell) . . . 168 ,,
4. Beinn Bhrotain (over south shoulder of Carn na Drochaide) . 355 ,,
5. Cairn Toul—south shoulder (over Bruach Mohr) . . . 130 ,,
   (The summit is not visible by 279 feet)
6. Tap o'Noth (over Knock Saul) . . . . . 237 ,,
7. Hill of Mormond—west top (over Hill of Dens) . . . 51 ,,
   ,, east top (over west shoulder of hill of Dudwick) . . . . . 54 ,,
8. Hill of Garvock (over west shoulder of Clochna Hill) . . 80 ,,

So far, we have been dealing with the visibility of objects at higher elevations than the point of observation. Let us now work out two or three examples where the objects are at a lower level.

(*a*) *Can the Blue Hill be seen from Morrone?*

On referring back to page 29 it will be seen that the reduced levels are :—

Morrone	Knockie Branar	Blue Hill
Feet	Feet	Feet
949	805	0

And the distance from Morrone to Knockie Branar is $16\frac{3}{4}$ miles, Knockie Branar being 144 feet lower, per reduced levels. Therefore, on the line of vision being produced over Knockie Branar to the Blue Hill, the total fall would be

$$\frac{50 \text{ miles}}{16\frac{3}{4} \text{ ,,}} \times 144 = 430 \text{ feet}$$

This, deducted from the reduced level of Morrone, 949 feet

less 430 ,,

brings out that the Blue Hill is not visible from Morrone by 519 feet

(*b*) *From the Blue Hill, 480 feet above sea level, how far off will a ship be when its topmast flag, 75 feet high above sea level, is just seen over the horizon?*

Per table, the sea horizon for 480 feet is distant . 29½ miles
,, 75 feet beyond the horizon . . . 11½ ,,

Distance from the Blue Hill to the ship . . . 41 miles

(*c*) *Seen from the top of Mount Keen, does the top of the Blue Hill appear to be above or below the visible sea horizon?*

Mount Keen is 33 miles from the Blue Hill, and being 3077 feet high it is visible, per table, at a distance of about 74½ miles from its sea horizon. From the same point (the Mount Keen sea horizon), the distance to the Blue Hill is 74½ − 33 = 41½ miles, and per table, would be seen if it were . . . . . 957 ft. high
But the Blue Hill being only . . . . 480 ,, ,,

it is therefore not visible above the horizon by . . . 477 ft.

Accompanying this article is a diagram shewing a section in a straight line from Morrone to the Blue Hill, and continued out in the same line to beyond the sea horizon. The hill-tops, etc., with the elevations taken from the Ordnance Survey levels, are plotted above a curved datum line, which represents the curvature of the earth corrected for the effects of refraction; and the heights are exaggerated to the extent of about 42 times greater than the lengths. One may not be prepared to vouch that the principle of largely increasing the scale for the heights which universally prevails in engineering sections with a straight datum line (the combined effects of curvature and refraction being eliminated), can be applied with mathematical accuracy to this section, having its datum line curved—not quite, but very nearly—to the circumference of a circle; but this may be said—that the method is sufficiently correct for all practical purposes, and that the present writer has for a considerable time used it as a rapid and graphic way of ascertaining whether certain distant objects can be seen or not, subsequently, if need be, verifying its accuracy by calculation.

Some one, not unnaturally, may ask why the line of vision from the Blue Hill to Morrone is not shown curved downward to the extent of one-sixth of the

DIAGRAM

Section in straight line from the Blue Hill to Morrone

Prepared for the Cairngorm Club Journal by Gordon Jenkins C.E.

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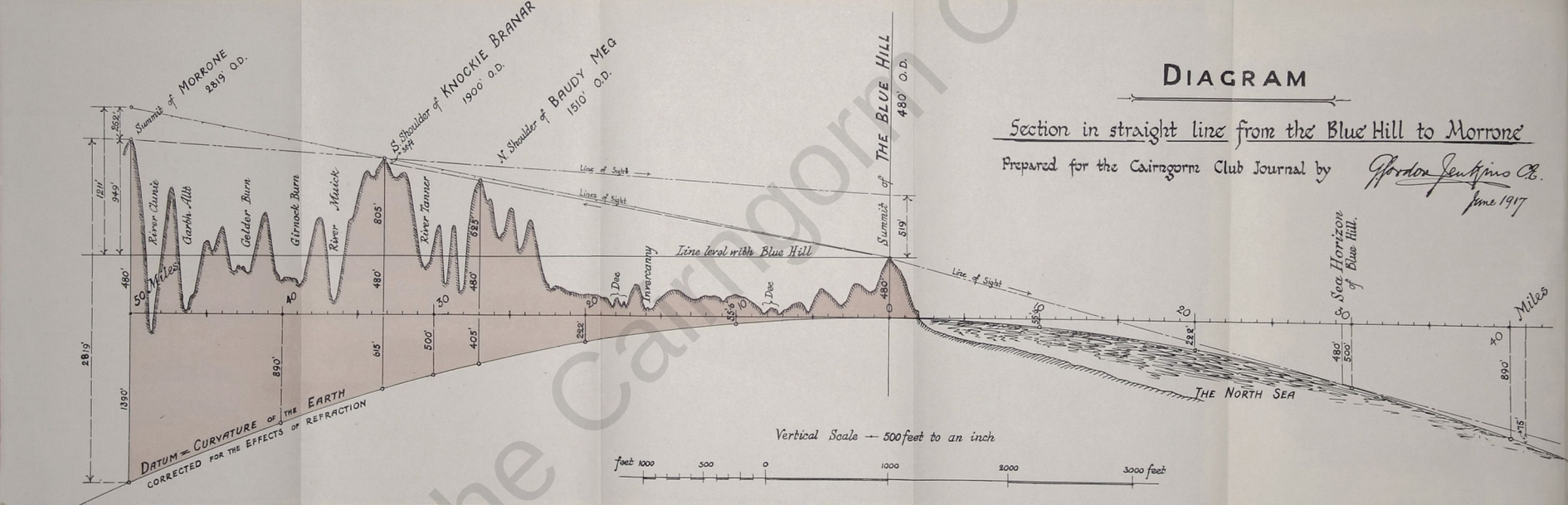

earth's curvature, or 278 feet, for the effect of refraction. The answer is that, as above noted, the datum line is corrected to the necessary extent, so that, for convenience, a straight line of sight can be used in the diagram for determining what summits intervene to obscure the view.

If it is wished by those of an inquiring mind to go into the calculations herein given, this diagram, with its detailed dimensions, will enable the various stages of the process to be followed, and will also enable the principles enunciated to be applied to other cases. Apart from that, however, the section pictorially indicates how the effect of the earth's curvature, partly neutralised by the effect of refraction, lowers the visibility of distant objects, this lowering effect, in the case of Morrone at 50 miles from the Blue Hill, being to the extent of 1390 feet compared with a level line as a tangent to the earth's surface at the latter point.

The state of perfection to which science has brought the art of measuring heights and distances may be inferred from the accuracy of the long-range gunnery on land and sea during the present war. Objects—say, 20 miles away—can be accurately hit although they are invisible to the gunner, whose direction of fire may be obtained only from a telephone message or a "wireless" from a flying machine, giving the position (on a map divided into numbered squares) of the spot required to be struck. Not only do the questions discussed in this article come into play, but also the angle of fire horizontally and vertically, as well as the trajectory of the bullet or shell.

This contribution touches only the fringe of the complex calculation of the diameter of the earth, etc., on which the data of the formula herein given are based; but if it serves to direct attention to the general principles of a subject of much practical importance in mountaineering, and helps to make them clear, even to a limited extent only, the author will be rewarded for any little trouble he has taken in placing the matter before the

members of the Cairngorm Club. He would like to add that his thanks are due to Mr. J. A. Parker, C.E., for the great trouble he took in revising the rough draft of this article, and for the valuable alterations he suggested towards putting it in the shape it now takes; and also to Mr. William W. Fyvie, Aberdeen University, for kindly checking the figures dealing with Morrone.

TABLE OF HEIGHTS AND DISTANCES CORRECTED FOR CURVATURE AND REFRACTION.

Distance	Height	Distance	Height	Distance	Height
Miles	Feet	Miles	Feet	Miles	Feet
1	.556	31	534	61	2069
2	2.22	32	569	62	2137
3	5.00	33	605	63	2207
4	8.90	34	643	64	2277
5	13.90	35	681	65	2349
6	20.02	36	721	66	2422
7	27.24	37	761	67	2496
8	35.58	38	803	68	2571
9	45.04	39	846	69	2647
10	55.60	40	890	70	2724
11	67	41	935	71	2803
12	80	42	981	72	2882
13	94	43	1028	73	2963
14	109	44	1076	74	3045
15	125	45	1126	75	3127
16	142	46	1176	76	3211
17	161	47	1228	77	3296
18	180	48	1281	78	3383
19	201	49	1335	79	3470
20	222	50	1390	80	3558
21	245	51	1446	81	3648
22	269	52	1503	82	3738
23	294	53	1562	83	3830
24	320	54	1621	84	3923
25	347	55	1682	85	4017
26	376	56	1744	86	4112
27	405	57	1806	87	4208
28	436	58	1870	88	4305
29	468	59	1935	89	4404
30	500	60	2002	90	4503

NOTE.—The distances are in statute miles of 5280 feet, from the point of observation to the sea horizon, corrected for the effects of refraction. The heights are the heights of the points of observation in feet above sea level.